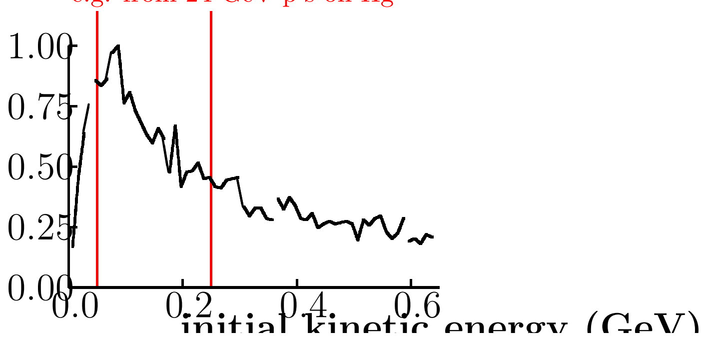
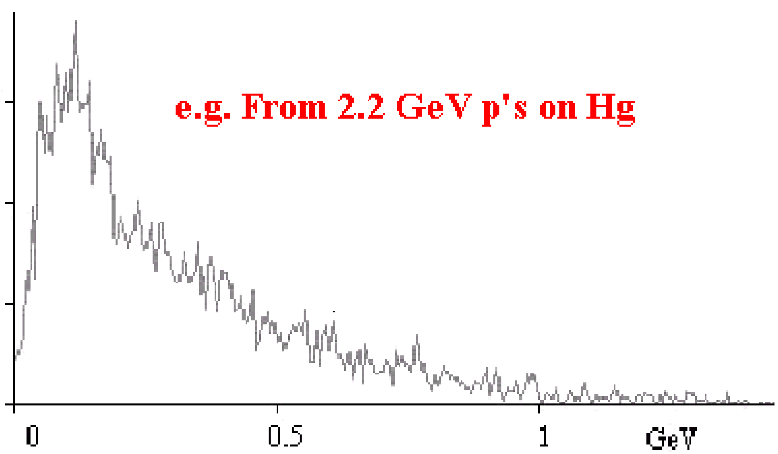
1 Pion Capture

1.0.1 Initial KE Distribution

e.g. from 24 GeV p's on Hg





- similar distributions
- Reasonable ≈50-250 MeV
- $\sigma_{p\perp}$≈150 MeV/c

1.1 Magnetic Horn Capture

1.1.1 Horn theory

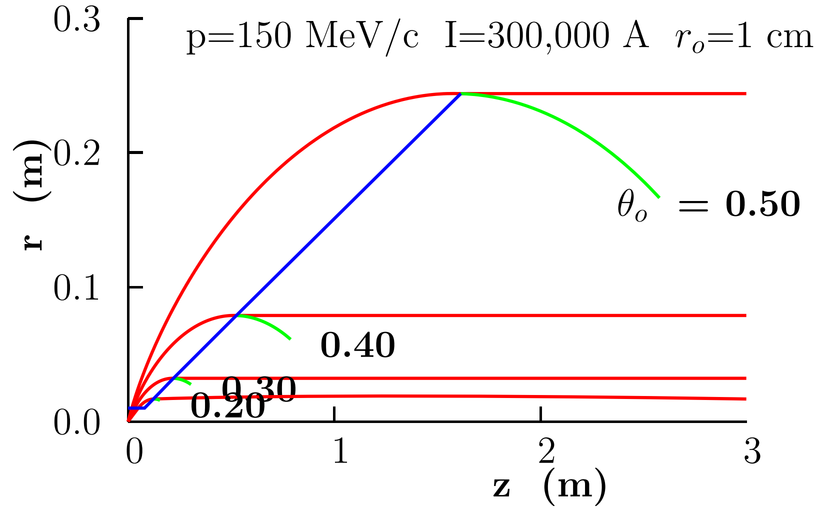
Outside an axial conductor

$$B = \frac{\mu_o\, I}{2\,\pi}\,\frac{1}{r}$$

Bending:

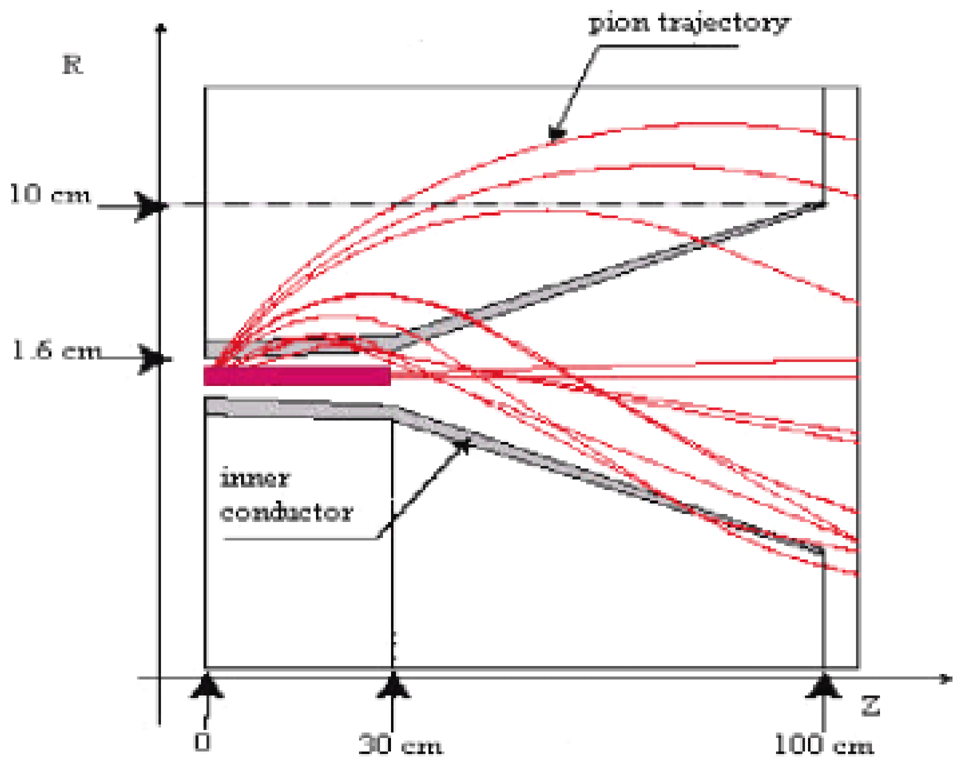
$$\frac{d\theta}{ds} = \frac{B\ c}{(p)}$$

Minimum radius set by inward forces. Find exit shape to focus mom=p:



1.1.2 Example

CERN Design



1.2 Solenoid Capture

In the transverse plane:

$$r = \frac{(p_\perp)}{c\ B}$$

For particles generated in a thin target on the axis, inside a solenoid of inside radius R, the maximum transverse momenta captured will be:

$$(p_\perp(max)) = \frac{c\ B_z\ R}{2} \tag{1}$$

e.g. For a 20 T solenoid of 8 cm radius, (These are the dimensions of an existing resistive solenoid at FSU)

$$p_\perp(max) \; = \; 240 MeV/c$$

Contains å80% of π's below 250 MeV

1.3 Adiabatic Matching

The match between a target capture Solenoid and a decay channel solenoid can be made, with negligible loss, by gently tapering the magnetic field[1] .

The condition for "gentleness" is that $d\beta/\beta$, is small in a distance equal to the current β:

$$\frac{d\beta}{\beta} \; \ll \; \frac{dz}{\beta}$$

or

$$\frac{d\beta}{dz} \; = \; \epsilon \; \ll \; 1$$

Since $\beta \; \propto 1/B_{solenoid}$:

$$\frac{d(1/B)}{dz} \; = \; \epsilon \; \ll \; 1$$

which gives:

$$B(z) \; = \; \frac{B_o}{1 \; + \; k \; z} \tag{2}$$

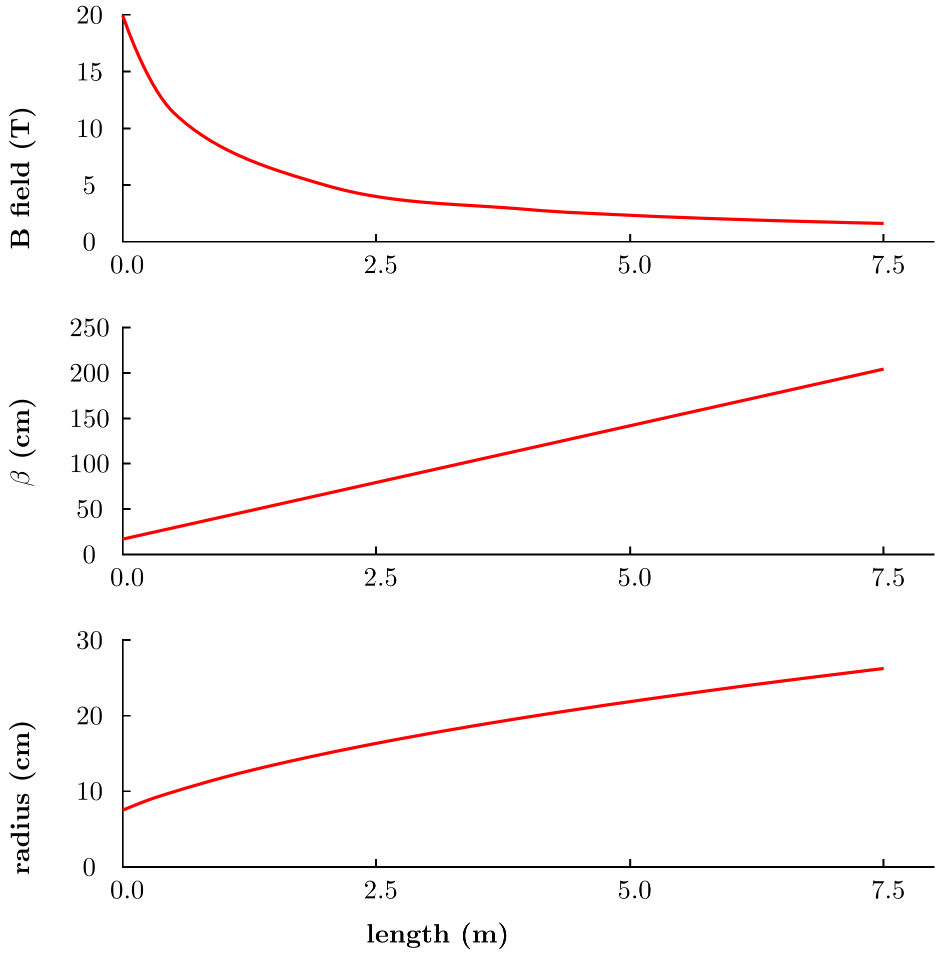
where

$$k \; = \; \epsilon \, \frac{B_o \; c}{2 \; (pc/e)} \tag{3}$$

Note that the B drops initially very fast, corresponding to the short β's at the high initial field, but falls much slower at the lower later fields where the β's are long.

For a taper from 20T to 1.25T at momenta less than 1 GeV and $\epsilon \; = \; .5$, the taper length should be approximately 6 m.

[1] R. Chehab, J. Math. Phys. 5 (1978) 9.



1.4 Phase Rotation

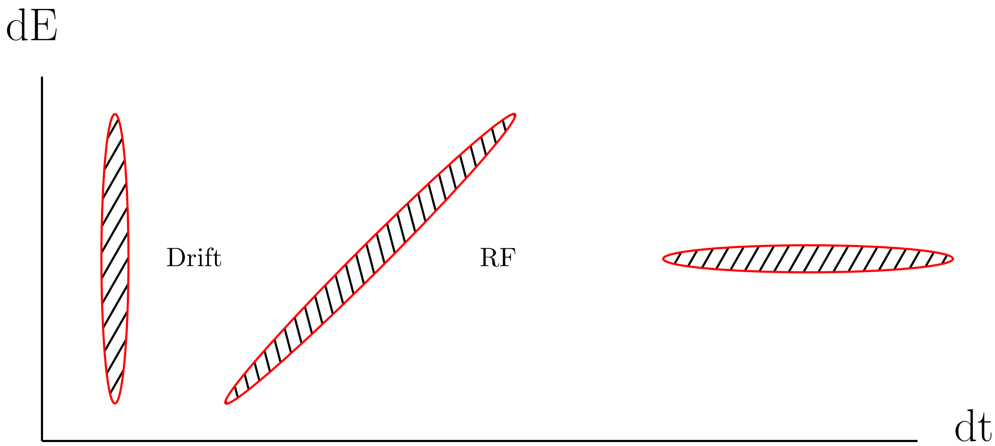
1.4.1 Introduction

- Initial pions have rms dp/p ≈ 100%
- rms Acceptance of cooling ≈ 8%

Phase Rotate

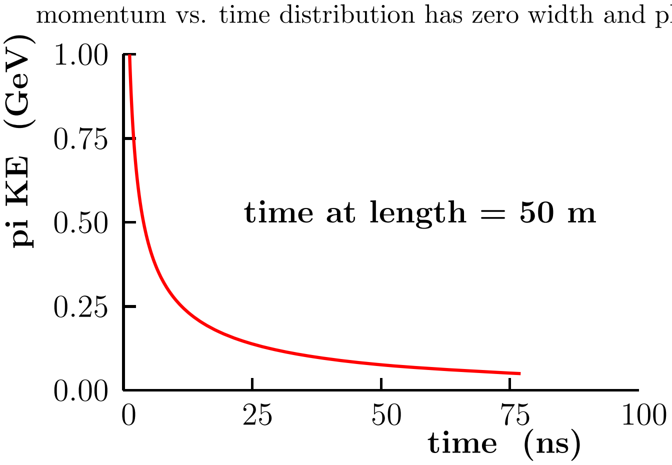
- Increase dt

- Decrease dE

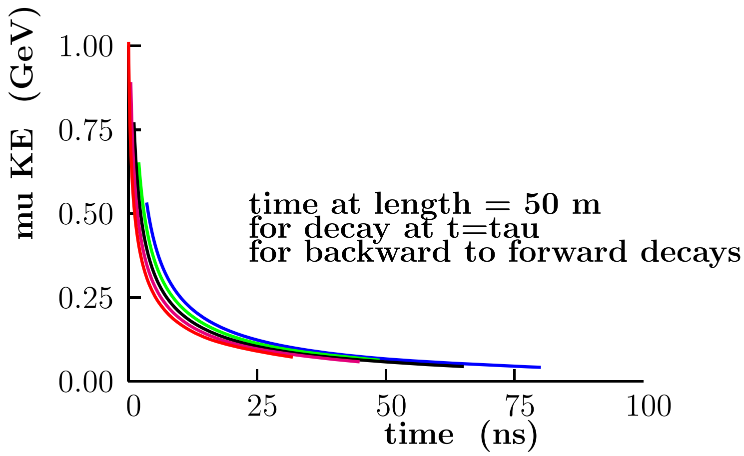


1.4.2 Time Jitter from Pion Decay

If the p bunch had zero length, and there was no decay, then after a drift the momentum vs. time distribution has zero width and phase rotation is ideal.



But since the pions decay to muons ($\pi \rightarrow \mu + \nu$) there is a spread from the random pion decay angle and decay position:



Decay in Center of Mass

$p_\mu \approx 30$ MeV/c $\approx m_\pi - m_\mu$

Isotropic, so

$\dfrac{dn}{dp_{\mu z}}$ is flat from -30 to 30 MeV/c

$$E_\mu \;=\; \sqrt{p_\mu^2 + m_\mu^2} \;\approx m_\mu$$

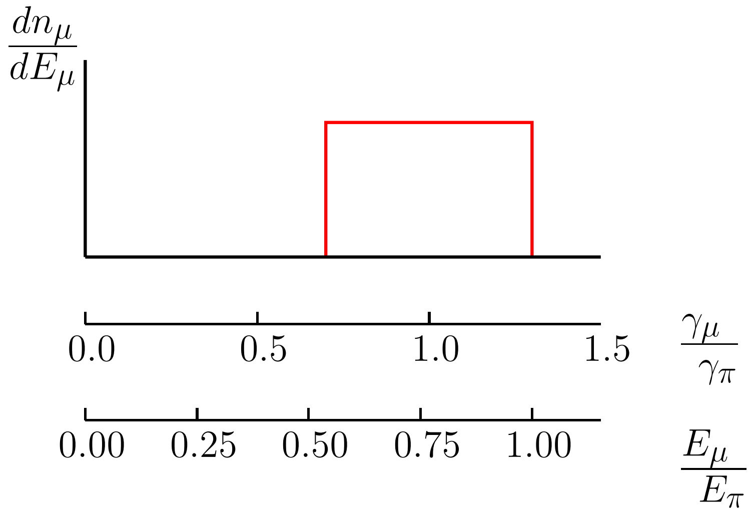
Lorentz Boost to velocity of initial π

$$\gamma_\pi \;=\; \frac{\mathrm{KE} + m_\pi}{m_\pi}$$

$$\beta_\pi \;=\; \sqrt{1 - \frac{1}{\gamma_\pi^2}}$$

$$E_\mu(\text{final}) \;=\; \gamma_\pi \quad E_\mu(\text{c of m}) \;+\; \beta_\pi\gamma_\pi \quad p_z(\text{c of m})$$

$$\gamma_\mu(\text{final}) \;\approx\; \gamma_\pi \quad \pm \quad \beta_\pi\gamma_\pi \quad \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$



$$< \gamma_\mu > = \gamma_\pi = \gamma$$

$$\Delta\gamma_\mu = \pm \beta\gamma \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$

$$\Delta\beta_\mu \approx \frac{d\beta}{d\gamma} \Delta\gamma_\mu = \frac{1}{\gamma^3\beta} \beta\gamma \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$

$$\Delta\beta_\mu = \frac{1}{\gamma^2} \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$

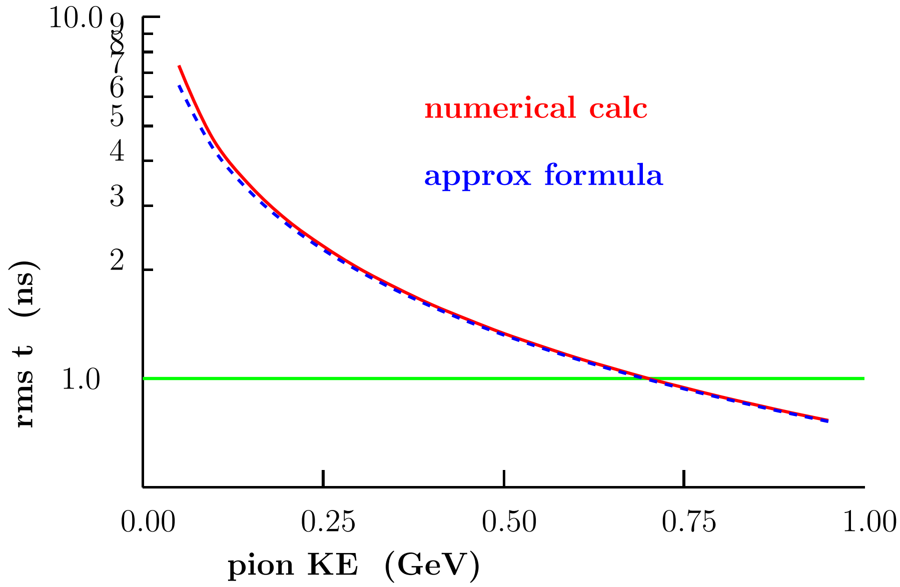
If decay occured at distance $\ell = \beta c\gamma\tau$ then Δt between forward and backward cases:

$$\Delta t \approx \frac{d}{d\beta} \left(\frac{L}{\beta c}\right) \Delta\beta = \frac{1}{\beta^2 c} \beta c\gamma\tau \, \Delta\beta$$

$$\Delta t \approx \frac{\tau}{\beta\gamma} \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$

The rms spread of a uniform distribution = $\sqrt{1/3} \times$ max, and the rms of the exponential is $=\sqrt{2} \times \tau$

$$\sigma_t \approx \sqrt{\frac{2}{3}} \frac{\tau}{\beta\gamma} \left(\frac{m_\pi - m_\mu}{m_\mu}\right)$$



Conclusion on jitter from decay

If we capture muons from 50 to 250 MeV, the average $KE_\pi \approx$ 190 MeV, where $\sigma_t \approx$ 2.7 ns. If we want the broadening from the proton σ_t to be $< 10\%$ then σ_t(p beam)<1.2 (nsec). For a 40% effect, it could be 2.7 nsec.

1.4.3 Phase Space Conservation

For initial $\Delta E = 200$ MeV (full width)
$\times\ \delta t = 4$ nsec (rms) (time is set by fluctuations in decay)

If final δE/E 8% (rms) at 200 MeV (δE=16 MeV (rms)):

$$\Delta t(final) \;=\; \frac{200(full) \times 4(rms)}{16(rms)} \;=\; 50nsec(full)$$

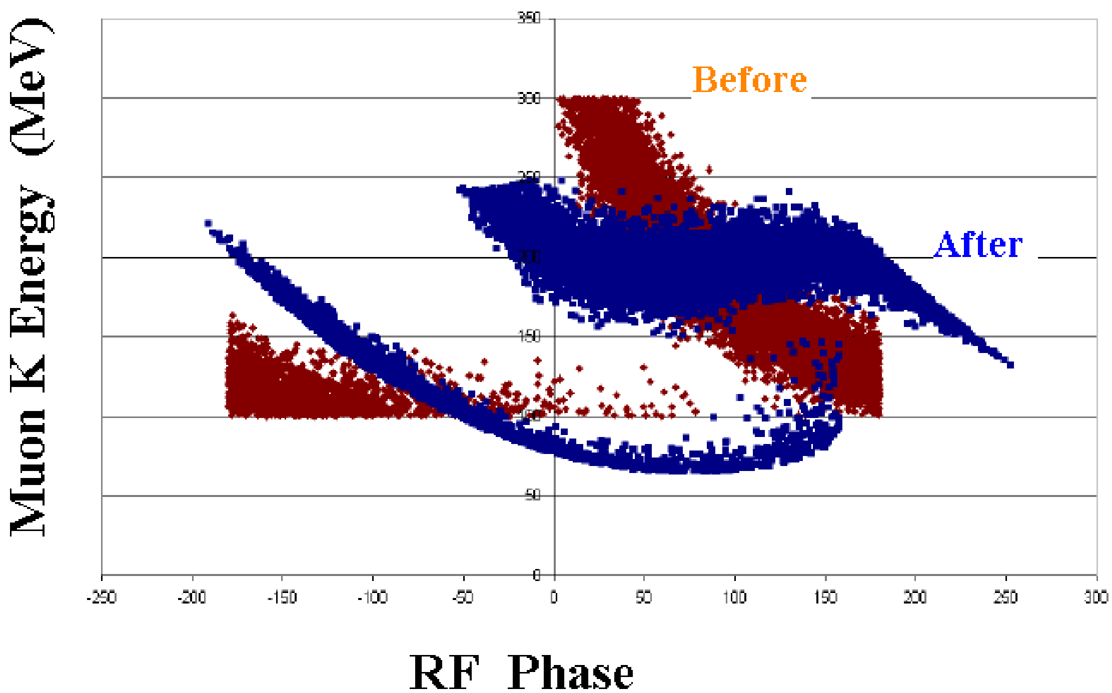
To capture and accelerate this we need
frequency $\ll$ 1/50(nsec), i.e. $\ll$ 20 MHz

- KEK: 5 MHz which would allow only low gradients.
- CERN: 44 or 88 MHz
- PJK: 30 MHz but got dp/p ≈15 %

1.4.4 Examples without re-bunching

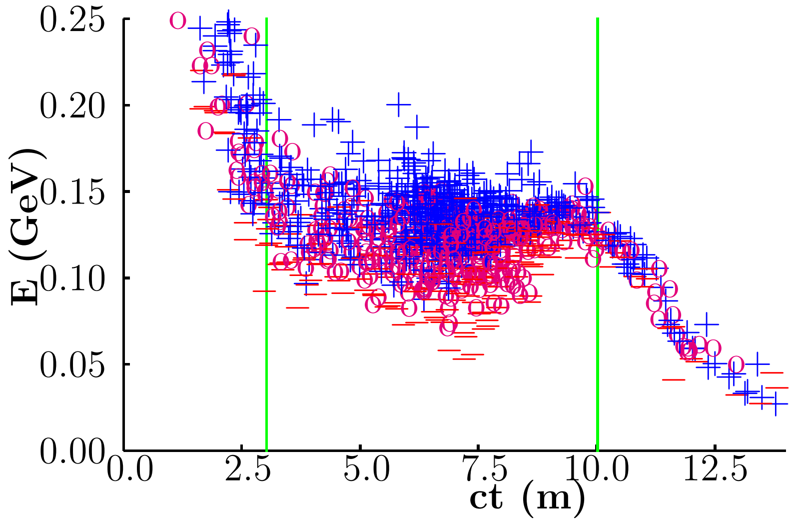
e.g. CERN

- 30 m decay channel
- 30 m 2 MV/m 44 MHz RF
- Captures ≈120-300 MeV
- Gives ≈4 m long bunch
- and ≈± 5%



e.g. PJK

	Len m	freq MHz	Grad MV/m
Drift	6		
RF	12	40	6
RF	24	30	5
RF	5	45	6



- ≈6 m long bunch
- ≈12 % dE/E

1.4.5 Examples with Re-Bunching

Alternative allowing higher frequencies:

Re-bunching increases dE/E by ≈4 ×
So require dE/E ≈2% before re bunching
And Δt ≈50 nsec ×4 ≈ 200 nsec

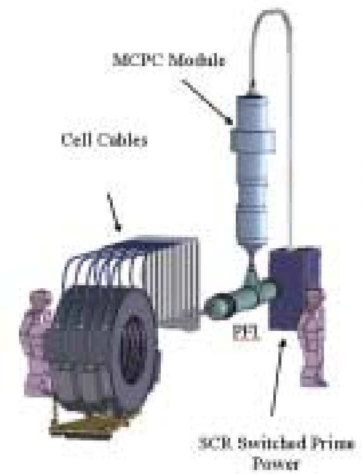
US Study 1 had ≈150 nsec
US Study 2 had ≈300 nsec

Too long for conventional rf,

Use Induction Linacs

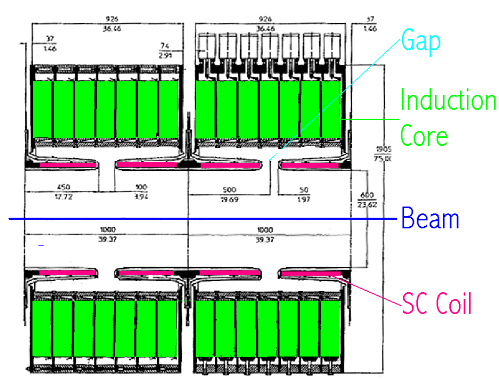
- pulses 50-500 nsec
- Grad's ≈1 MV/m

1.5 Induction Linacs



2m Section

95 cm radius similar to ATA or DARHT but Superconducting inside coil



1.5.1 Example of Single Linac PR

US Study 1

- Energy spread non uniform
  "Distorted"
- dp/p rms ≈ 6%
- → 18% after bunching
- particles lost

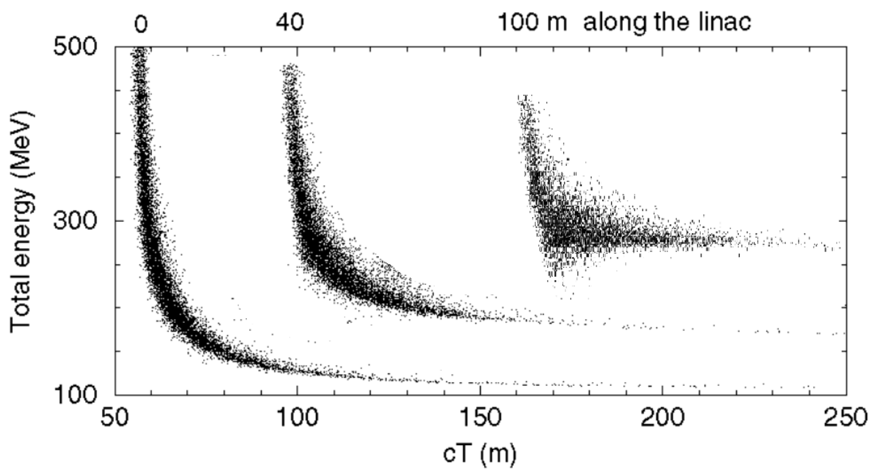
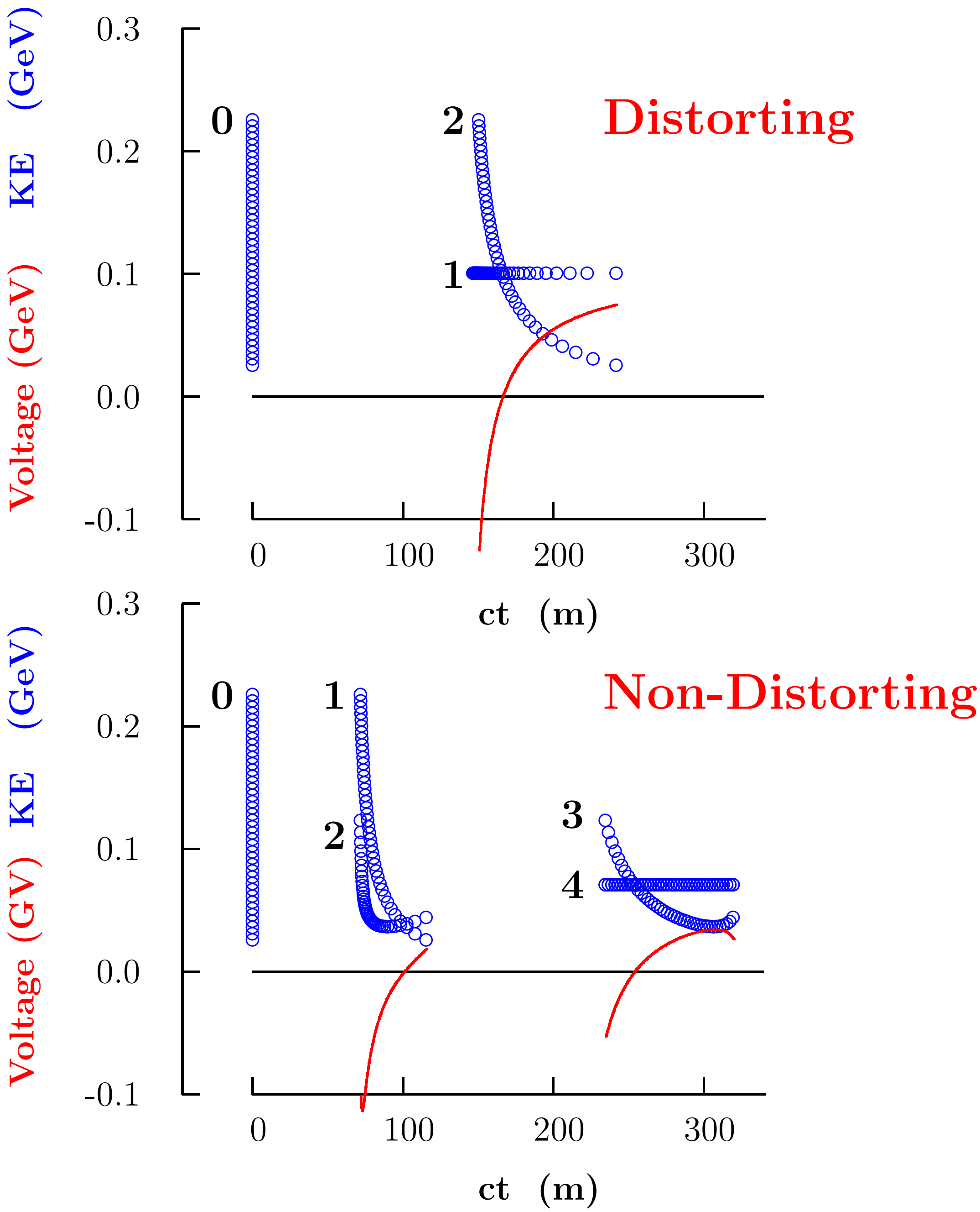


Figure 6: Beam distributions in E-cT phase space along the induction linac. Distributions from L = 0, 20, 60, and 100 m are shown.

1.5.2 Non-Distorting Phase Rotation

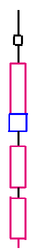
. MUC-114



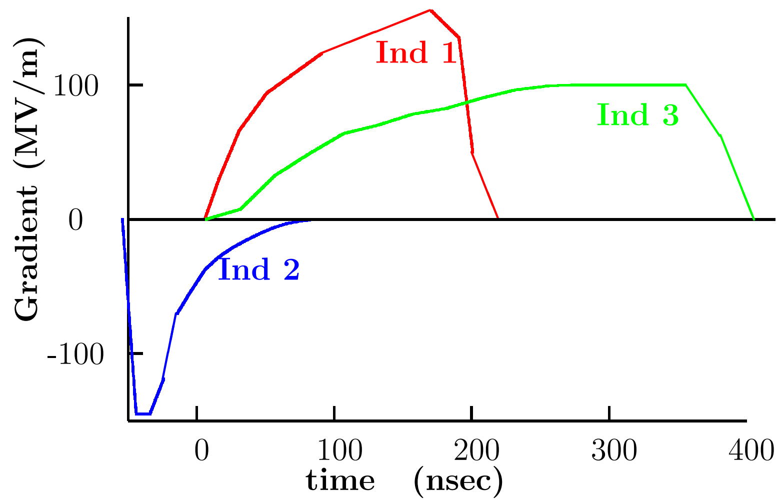
1.5.3 Example of Non-Distorting

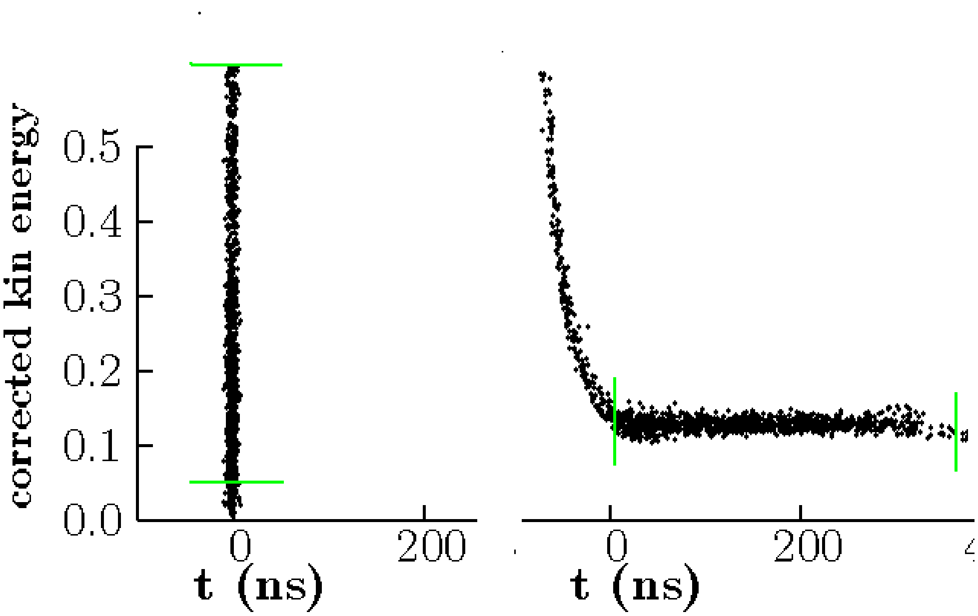
Study 2 2-3 Linacs

1. 30 m Drift
2. Induction Linac to modify E vs t
3. Second drift ($\approx$ 100 m)
4. 2nd Induction Linac to reduce dE/E



Hg Target	**(.45 m)**
Induction #1	**(100 m)**
Mini Cooling	**(3.5 m H_2)**
Induction #2	**(80 m)**
Induction #3	**(80 m)**





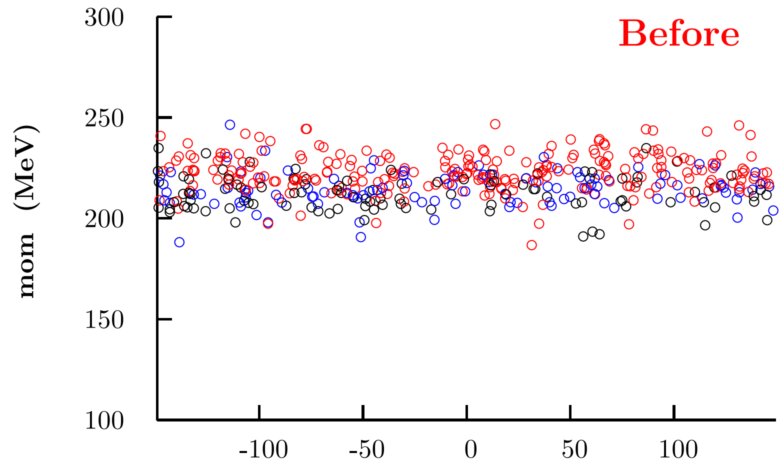
- Energy spread more uniform
- dp/p rms $\approx$ 3%
- OK for bunching

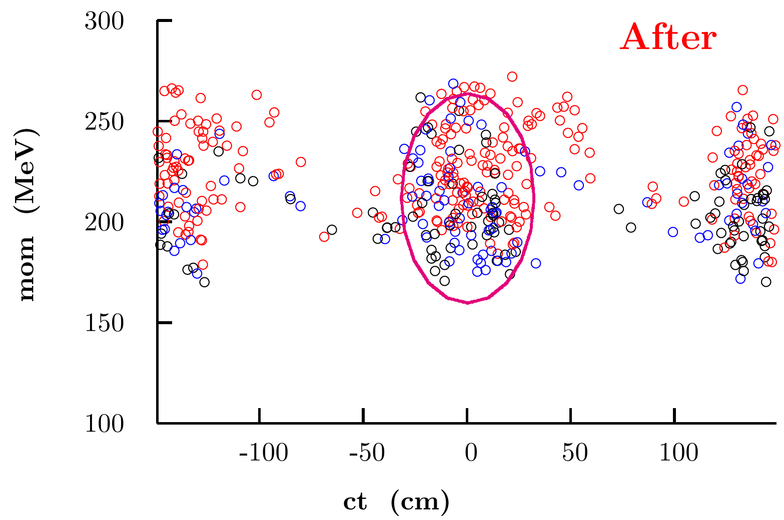
1.6 RF Buncher

Three stages:

stage		len m	400 MHz MV	200 MHz MV
1	RF	2.75	-2.38	9.55
	Drift	22		
2	RF	5.5	-4.46	17.9
	Drift	8.25		
3	RF	8.25		35.8
	Drift	5.5		

Similar to Study 1



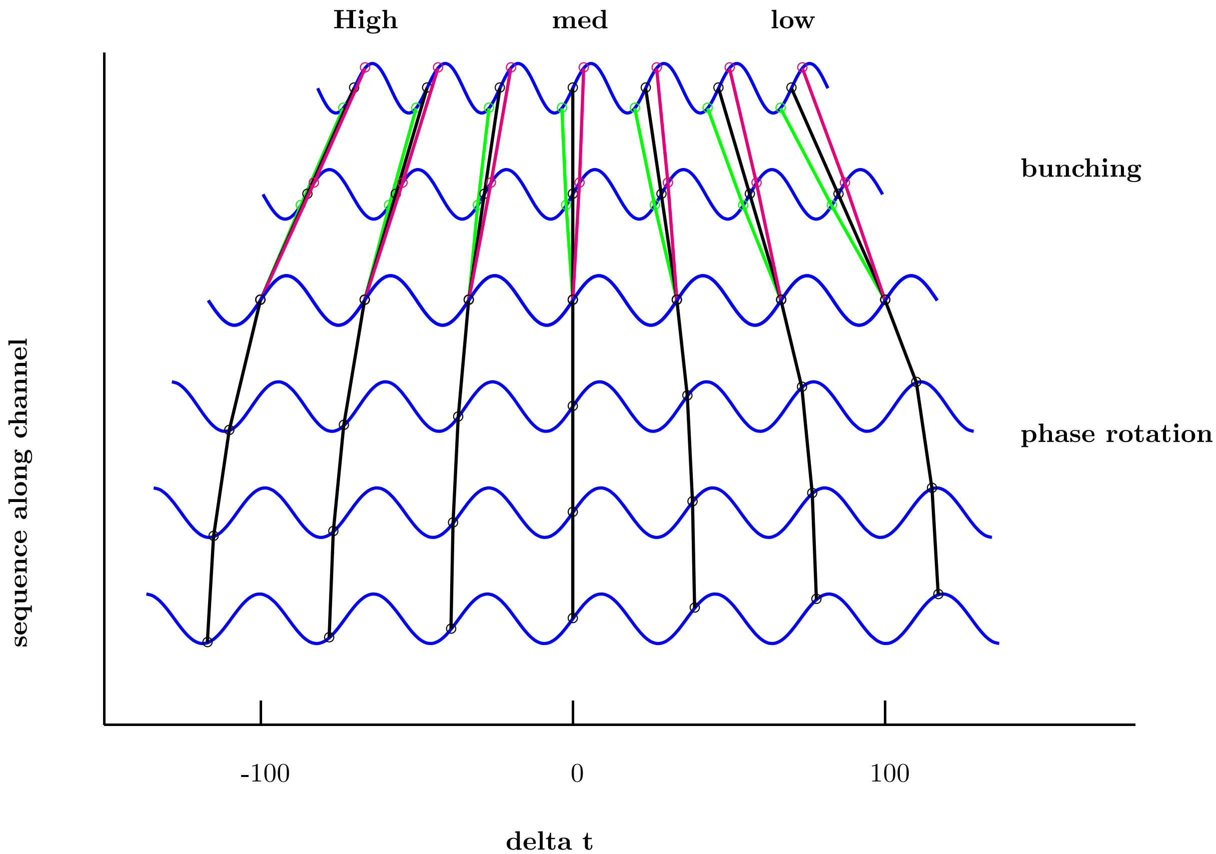


Bunched Phase Rotation

1. Drift
2. Bunch
3. Rotate with high freq. rf

vs. Conventional

1. Drift
2. Rotate with induction linac
3. Bunch



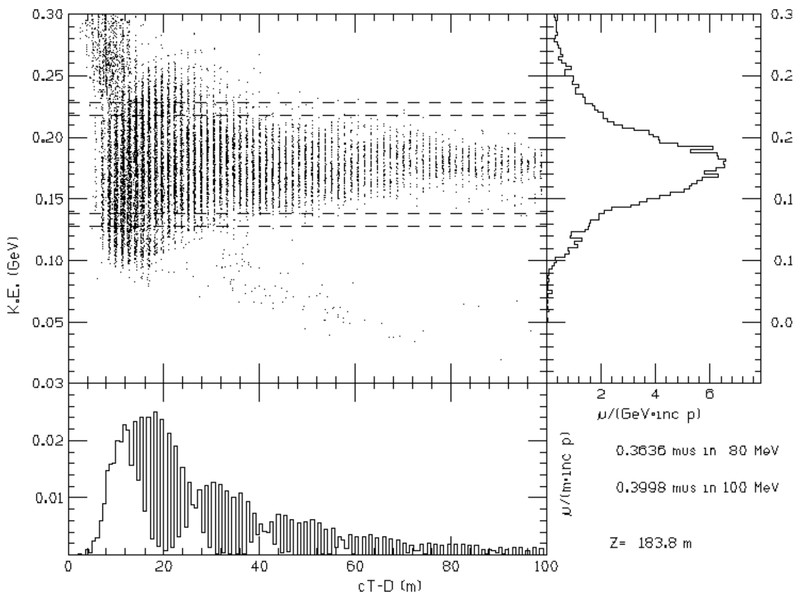


Figure 7: Muon distribution in (E, t)-space along with marginal distributions for 38 vernier (d=0.16) cavities followed by 23 (matched) fixed frequency cavities *generated with* ICOOL *program.* N_b=20 in buncher part. Plots and numbers quoted are based on 188 000 incident protons.

Compare with conventional

1. Inevitably Distorting
2. Probably less efficient for one sign
3. But both signs rotated
4. Much less cost than induction